



On Properties of Zero Points and Poles of K-Bianalytic Functions

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Authors' contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

Article Information

DOI: <https://doi.org/10.9734/jamcs/2024/v39i111946>

Open Peer Review History:

This journal follows the Advanced Open Peer Review policy. Identity of the Reviewers, Editor(s) and additional Reviewers, peer review comments, different versions of the manuscript, comments of the editors, etc are available here: <https://www.sdiarticle5.com/review-history/126347>

Received: 08/09/2024

Accepted: 11/11/2024

Published: 18/11/2024

Original Research Article

Abstract

In this paper, we first prove that the set of zero points of a nonzero K-bianalytic function $f(z) = \bar{z}(k)\phi_1(z(k)) + \phi(z(k))$, $z \in D$, is not a region and the set of the second zero points has no accumulated point. Second, a sufficient and necessary condition is given for a K-bianalytic function to have a zero arc which has a parameter equation $\bar{z}(k) = \gamma(z(k))$ where γ is an analytic function in a region $D(k)$. Finally, the traits of a K-bianalytic function which has a zero arc, even straight, one of whose ends is a (c_1, c_2) -th pole at $z = 0$, are discussed. Some examples are also shown for our topic.

Keywords: Zero points; poles; K-bianalytic functions.

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Cite as: Ni, Junyou, and Hongliang Li. 2024. "On Properties of Zero Points and Poles of K-Bianalytic Functions". *Journal of Advances in Mathematics and Computer Science* 39 (11):160-67. <https://doi.org/10.9734/jamcs/2024/v39i111946>.

1 Introduction

Zhang [1] gave the definition of K -analytic functions. Zhang and his coauthors extended some properties of analytic functions to K -analytic functions in the theories of integral, Laurent series expansion, residual theorem, fractional linear transformation, conformal mapping, and so on. See [2]-[8]. Sang and Li [9] studied the mean value theorems of K -analytic functions. Lin and Xu [10] investigated Riemann problem of (λ, k) bi-analytic functions.

There are many works to study the properties of bianalytic functions. For example, Zhu, Huang, Liu and Zhu [11] gave the distribution of nonisolated zero points, properties of mapping, classification of isolated singular points of bianalytic functions. Fu [12] summarized some properties of analytic and bianalytic functions in the dissertation for master's degree. Wang, Huang and Liu [13] researched the properties of bianalytic functions with zero arc at a pole.

Li and Liu [14] naturally put forward the concept of K -bianalytic functions and investigated Cauchy theorem, Cauchy integral formula, power series expansion, Fourier series expansion of K -bianalytic functions. Hitherto, a lot of properties of K -bianalytic functions have not yet been studied and thus it is necessary to continue to look into the attributes of K -bianalytic functions.

In this paper, we mainly explore the properties of zero points and poles in K -bianalytic functions, which generalize the corresponding results of [11] and [13] in bianalytic functions.

Definition 1.1. [1] The forms of complex number as $x + iky$ ($k \in \mathbb{R}, k \neq 0$) are called K -complex number of $x + iy$, denoted by $z(k)$.

Definition 1.2. [1] Let the function $f(z)$ be defined in a neighborhood of z_0 . If

$$\lim_{\Delta z(k) \rightarrow 0} \frac{\Delta f}{\Delta z(k)} = \lim_{z \rightarrow z_0} \frac{f(z) - f(z_0)}{z(k) - z_0(k)}$$

exists, then we call that $f(z)$ is K -differential at z_0 , the limit is the K -derivative of $f(z)$ at z_0 , denoted by $f'_{(k)}(z_0)$ or $\left. \frac{df(z)}{dz(k)} \right|_{z=z_0}$, i.e.,

$$f'_{(k)}(z_0) = \left. \frac{df(z)}{dz(k)} \right|_{z=z_0} = \lim_{z \rightarrow z_0} \frac{f(z) - f(z_0)}{z(k) - z_0(k)}.$$

If $f(z)$ is K -differential at each $z \in D$, then the second K -derivative of $f(z)$ at $z_0 \in D$ is defined as

$$f''_{(k)}(z_0) = \left. \frac{df'_k(z)}{dz(k)} \right|_{z=z_0} = \lim_{z \rightarrow z_0} \frac{f'_k(z) - f'_k(z_0)}{z(k) - z_0(k)}.$$

Similarly, the n th K -derivative $f^{(n)}_{(k)}(z)$ can be defined as the same way.

Definition 1.3. [1] If $f(z)$ is K -differential in a region D , we say that $f(z)$ is analytic in D ; If $f(z)$ is K -analytic in a neighborhood of z_0 , then we say that $f(z)$ is K -analytic at z_0 .

Definition 1.4. [14] Let the function $f(z)$ have the second partial derivative $\frac{\partial^2 f(z)}{\partial \bar{z}(k)^2}$ in a region D . If $\frac{\partial^2 f(z)}{\partial \bar{z}(k)^2} = 0$ for any $z \in D$, then $f(z)$ is called a K -bianalytic function in D .

Denote $D(k) = \{\xi(k) | \xi \in D\}$ if D is a set of \mathbb{C} .

Lemma 1.1. [14, Theorem 1] *If function $f(z)$ is a bianalytic function in a region D , then the following is established*

$$f(z) = \bar{z}(k)\phi_1(z(k)) + \phi_2(z(k)), \quad z \in D$$

where $\phi_1(z)$ and $\phi_2(z)$ are arbitrary analytic functions in $D(k)$.

Similar to the definition of the n th zero point of bianalytic function in [11], the n th zero point of a K -bianalytic function $f(z)$ is defined as follows.

Definition 1.5. Let $f(z)$ be a K -bianalytic function in a region D , $n \geq 1$, $z_0 \in D$. If $f(z_0) = 0$ and $\frac{\partial^{i+j}}{\partial^i z(k) \partial^j \bar{z}(k)} f(z) = 0$ for any $0 < i + j \leq n - 1, i, j \in \mathbb{N}$ and there exists $s, t \in \mathbb{N}$ such that $s + t = n$ and $\frac{\partial^n}{\partial^s z(k) \partial^t \bar{z}(k)} f(z) \neq 0$, then z_0 is called a n th zero point of $f(z)$.

2 The Main Results

The zero points of K -bianalytic function are not definitely isolated. For example, the zero points of

$$w(z) = z(k)\bar{z}(k) - 1, \quad w(z) = z(k) - \bar{z}(k)$$

are the ellipse $x^2 + k^2 y^2 = 1$ and the real axis, respectively, but they are not zero functions. Although zero points of K -bianalytic functions are not non-isolated, the distribution of zero points is not very wide.

Theorem 2.1. *Let $w(z)$ be a nonzero K -bianalytic function in a region D . Then the set of zero points is not a region.*

Proof. Let $w(z) = 0, z \in \sigma$, where σ is a subregion of D . By Lemma 1,

$$w(z) = \bar{z}(k)\phi_1(z(k)) + \phi_2(z(k)),$$

where $\phi_1(z)$ and $\phi_2(z)$ are arbitrary analytic functions in $D(k)$. If $\phi_1(k(z)) = 0$ for $z \in \sigma$, by the isolation of zero points of K -analytic functions [3], we know that $\phi_1(k(z)) = 0$ for $z \in D$ and thus $\phi_2(k(z)) = 0$ for $z \in D$. This contradicts the condition of the theorem. If $\phi_1(k(z)) \neq 0$ for $z \in \sigma$, then there exists $z_0 \in \sigma$ such that $\phi_1(k(z_0)) \neq 0$ and thus there is a neighborhood of $z_0, U(z_0) \in \sigma$, such that $\phi_1(k(z)) \neq 0$ for $z \in U(z_0)$. Since

$$w(z) = \bar{z}(k)\phi_1(z(k)) + \phi_2(z(k)) = 0, \quad z \in \sigma,$$

it follows that

$$\bar{z}(k) = -\frac{\phi_2(z(k))}{\phi_1(z(k))}, \quad z \in \sigma,$$

which is wrong obviously. □

Theorem 2.2. *The second zero points of a K -bianalytic function has no accumulated point.*

Proof. Suppose that

$$w(z) = \bar{z}(k)\phi_1(z(k)) + \phi_2(z(k)),$$

where $\phi_1(z)$ and $\phi_2(z)$ are arbitrary analytic functions in $D(k)$. If the set of the second zero points $z_n, n = 1, 2, \dots$, has a accumulated point $z_0 \in D$. In the light of

$$w(z_n(k)) = \phi_1(z_n(k)) = 0, \quad n = 1, 2, \dots,$$

we have that

$$\phi_2(z_n(k)) = 0, \quad n = 1, 2, \dots$$

Uniqueness of K -analytic function [3] gives that

$$\phi_1(z(k)) = \phi_2(z(k)) = 0, \quad z \in D,$$

which contradicts the fact that w is a nonzero function in D . □

Definition 2.1. If the points of an arc γ are zero points or the accumulated points of zero points of a K -banalytic function $w(z)$, then γ is called a zero arc of the K -banalytic function $w(z)$. If $w(z) = C$ for $z \in \gamma$ where C is a constant, then γ is called a constant arc of the K -banalytic function $w(z)$.

Theorem 2.3. Let the curve $\widehat{\gamma}$ have a parameter equation $\bar{z}(k) = \gamma(z(k))$ where γ is an analytic function in a region $D(k)$. Then $\widehat{\gamma}$ is a zero arc of a K -banalytic function in D $w(z) = \bar{z}(k)\phi_1(z(k)) + \phi_2(z(k))$ if and only if

$$\phi_2(z(k)) = -\gamma(z(k))\phi_1(z(k)), \quad z \in D. \tag{2.1}$$

Proof. Necessity. If

$$\phi_1(z(k)) = 0, \quad w(z) = 0, \quad z \in \widehat{\gamma},$$

then

$$\phi_2(z(k)) = 0, \quad z \in \widehat{\gamma}.$$

Therefore by uniqueness of K -analytic function [3] we know that

$$w(z) = \phi_1(z(k)) = \phi_2(z(k)) = 0, \quad z \in D.$$

If there exists $z_0 \in \widehat{\gamma}$ such that $\phi_1(z_0(k)) \neq 0$, $z_0 \in \widehat{\gamma}$, then there is a neighborhood of z_0 , $U(z_0) \in \sigma$, such that $\phi_1(z(k)) \neq 0$ for $z \in U(z_0)$. If $z \in \gamma \cap U(z_0)$, then

$$\begin{aligned} w(z) &= \bar{z}(k)\phi_1(z(k)) + \phi_2(z(k)) \\ &= \left(\bar{z}(k) + \frac{\phi_2(z(k))}{\phi_1(z(k))} \right) \phi_1(z(k)) \\ &= \left(\gamma(z(k)) + \frac{\phi_2(z(k))}{\phi_1(z(k))} \right) \phi_1(z(k)). \end{aligned}$$

Thus

$$\gamma(z(k)) + \frac{\phi_2(z(k))}{\phi_1(z(k))} = 0, \quad z_0 \in \widehat{\gamma} \cap U(z_0).$$

Uniqueness of K -analytic function [3] yields (2.1).

Sufficiency. Since $\phi_2(z(k)) = -\gamma(z(k))\phi_1(z(k))$, $\bar{z}(k) = \gamma(z(k))$, $z \in \widehat{\gamma}$, we have

$$\bar{z}(k)\phi_1(z(k)) + \phi_2(z(k)) = 0, \quad z \in \widehat{\gamma},$$

and thus

$$w(z) = \bar{z}(k)\phi_1(z(k)) + \phi_2(z(k)) = 0, \quad z \in \widehat{\gamma},$$

i.e., γ is a zero arc of w . □

Corollary 2.1. Under the assumptions of Theorem 2.3, $\widehat{\gamma}$ is a constant arc of a K -banalytic function $w(z) = \bar{z}(k)\phi_1(z(k)) + \phi_2(z(k))$ if and only if there exists a constant C such that

$$\phi_2(z(k)) - C = -\gamma(z(k))\phi_1(z(k)), \quad z \in D.$$

Example 2.1. Note that $\bar{z}(k) = z(k)$ is the parameter equation of the real axis. Thus by Theorem 2.3 a K -banalytic function $w(z)$ which takes a subarc of the real axis to be zero arc must be

$$w(z) = (\bar{z}(k) - z(k))\phi_1(z(k)),$$

where $\phi_1(z)$ is analytic on the real axis.

Definition 2.2. Let $w(z) = \bar{z}(k)\phi_1(z(k)) + \phi_2(z(k))$ be a K -banalytic function in a region D , where ϕ_1, ϕ_2 are analytic functions in $D(k)$ and $z = a$ is the c_i -th pole of $\phi_i, i = 1, 2$, respectively (if $z = a$ is a removable singular point of ϕ_i , then $z = a$ is called a 0-th pole of ϕ_i). If $0 \leq c_i < \infty (i = 1, 2)$ and $c_1^2 + c_2^2 \neq 0$, then $z = a$ is called (c_1, c_2) -th pole of $w(z)$.

Without loss of generality, we only need to discuss the behavior near $z = 0$ of $w(z)$. If $a \neq 0$, under the transformation $z(\zeta) = \zeta + a$, we can similarly investigate the behavior near $\zeta = 0$ of the function

$$\begin{aligned} w(z(\zeta)) &= \overline{\zeta + a}(k)\phi_1(z(\zeta)(k)) + \phi_2((z(\zeta))(k)) \\ &= \bar{\zeta}(k)\phi_1(z(\zeta)(k)) + [\bar{a}(k)\phi_1(z(\zeta)(k)) + \phi_2((z(\zeta))(k))]. \end{aligned}$$

Obviously, if $\phi_i, i = 1, 2$, have c_i -th poles of $\phi(z)$ at $z = 0$, respectively, then $\phi_i, i = 1, 2$, can be expressed by Laurent expansions as follows:

$$\phi_1(z) = \frac{a_{-c_1}}{z^{c_1}} + \frac{a_{-c_1+1}}{z^{c_1-1}} + \dots + a_0 + a_1z + \dots = \frac{1}{z^{c_1}}\psi_1(z); \tag{2.2}$$

$$\phi_2(z) = \frac{b_{-c_2}}{z^{c_2}} + \frac{b_{-c_2+1}}{z^{c_2-1}} + \dots + b_0 + b_1z + \dots = \frac{1}{z^{c_2}}\psi_2(z), \tag{2.3}$$

respectively. If $c_1 \geq 1$, then $a_{-c_1} \neq 0$ as well as c_2 does. The notations of (2.2) and (2.3) are used in the remaining part.

Theorem 2.4. Let a K -banalytic function

$$w(z) = \bar{z}(k)\phi_1(z(k)) + \phi_2(z(k))$$

have (c_1, c_2) -th pole at $z = 0$. If there is an arc γ with an end $z = 0$ such that

$$w(z) = 0, \quad z \in \gamma \setminus \{0\}, \tag{2.4}$$

then

$$c_1 = c_2 + 1, \quad |a_{-c_1}| = |b_{-c_2}|.$$

Proof. By contradiction. If $c_1 \leq c_2$, by (2.2) and (2.3) we have

$$w(z) = \frac{\bar{z}(k)z(k)^{c_2-c_1}\psi_1(z(k)) + \psi_2(z(k))}{z(k)^{c_2}}, \quad z \in \gamma \setminus \{0\}.$$

By (2.4), we have

$$\bar{z}(k)z(k)^{c_2-c_1}\psi_1(z(k)) + \psi_2(z(k)) = 0, \quad z \in \gamma \setminus \{0\}.$$

But

$$\lim_{z \rightarrow 0} (\bar{z}(k)z(k)^{c_2-c_1}\psi_1(z(k)) + \psi_2(z(k))) = b_{-c_2} \neq 0,$$

which is a contradiction. The similar method is suitable for explaining incorrectness of the case $c_2 < c_1 - 1$. Thus $c_1 = c_2 + 1$. In this case we obtain

$$w(z) = \frac{\bar{z}(k)\psi_1(z(k)) + z(k)\psi_2(z(k))}{z(k)^{c_1}}, \quad z \in \gamma \setminus \{0\},$$

which yields that

$$\bar{z}(k)\psi_1(z(k)) + z(k)\psi_2(z(k)) = 0, \quad z \in \gamma \setminus \{0\}. \tag{2.5}$$

So

$$\lim_{z \rightarrow 0, z \in \gamma} \left| \frac{\bar{z}(k)}{z(k)} \right| = \lim_{z \rightarrow 0, z \in \gamma} \left| -\frac{\psi_2(z(k))}{\psi_1(z(k))} \right| = \left| \frac{b_{-c_2}}{a_{-c_1}} \right| = 1.$$

□

Theorem 2.5. *Let a K -bianalytic function*

$$w(z) = \bar{z}(k)\phi_1(z(k)) + \phi_2(z(k))$$

have a (c_1, c_2) -th pole at $z = 0$. Then there exists a line segment η with an end $z = 0$, such that

$$w(z) = 0, \quad z \in \eta \setminus \{0\}, \tag{2.6}$$

if and only if there exists a neighborhood $U(0)$ of $z = 0$ such that

$$\frac{\phi_2(z(k))}{\phi_1(z(k))} = e^{i\theta_0} z(k), \quad z \in U(0),$$

where $\theta_0 = \arg \frac{b_{-c_2}}{a_{-c_1}}$.

Proof. Sufficiency. If $\frac{\phi_2(z(k))}{\phi_1(z(k))} = e^{i\theta_0} z(k)$, then there exists a line segment η with an end $z = 0$, such that (2.7) holds, where the line segment η satisfies the equation $y = \frac{1}{k} (\cot \frac{\theta_0}{2}) x$ for $x + iy \in \eta$.

Necessity. By assumptions and the proof of Theorem 2.4, we get $c_1 = c_2 + 1$ and (2.5) holds with γ replaced by η . Let the inclination of the line segment $\{\eta(k) = z(k) : z \in \eta\}$ is α . Hence there exists a deleted neighborhood $U^0(0)$ of $z = 0$ such that

$$\frac{\psi_2(z(k))}{\psi_1(z(k))} = -e^{-2i\alpha}, \quad z \in \eta \cap U^0(0).$$

The fact that $\frac{\psi_2(z(k))}{\psi_1(z(k))}$ is K -analytic, uniqueness of K -analytic function and (2.5) implies that there exists a neighborhood $U(0)$ of $z = 0$ such that

$$\frac{\phi_2(z(k))}{\phi_1(z(k))} = \frac{\psi_2(z(k))}{\psi_1(z(k))} z(k) = e^{i\theta_0} z(k), \quad z \in U(0).$$

□

Example 2.2. *Let K -bianalytic function be given by $w(z) = \frac{\bar{z}(k)+a_1z(k)+a_2z(k)^2+\dots+a_jz(k)^j+\dots}{z(k)^n}$, where $n \in \mathbb{N}^*$ and $a_i \in \mathbb{C}$. Then by Theorem 2.5 we know that there exists a line segment η with an end $z = 0$, such that*

$$w(z) = 0, \quad z \in \eta \setminus \{0\} \tag{2.7}$$

if and only if $a_1 = e^{i\theta}$, $\theta \in \mathbb{R}$ and $a_i = 0, i \geq 2$.

3 Conclusion

In the present paper, we extend some properties of zero points, zero arcs and poles of bianalytic functions to K -bianalytic functions.

Disclaimer (Artificial Intelligence)

Author(s) hereby declare that NO generative AI technologies such as Large Language Models (ChatGPT, COPILOT, etc) and text-to-image generators have been used during writing or editing of this manuscript.

Acknowledgement

This work was supported by the National Natural Science Foundation of China (11961056).

Competing Interests

Authors have declared that no competing interests exist.

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